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State synchronization of n-dimensional chaotic systems via the single-input robust adaptive sliding mode control

Hongyi Chen[2], Shihhung Kao[3], Kunchieh Wang[2], Huicun Shen[4], Chiching Yang[5,6]

Abstract. The novel single input, two-level robust adaptive sliding mode control is introduced to achieve the state synchronization for two the same type of n-dimensional nonlinear chaotic systems in this study. Unlike directly eliminated the nonlinear items by the developed control law in the literature, the proposed control law is not only required the known bounds of system uncertainties and external disturbances in prior but also with time varying feedback gains can compensate nonlinear dynamics of the synchronous error system. Meanwhile, these feedback gains are not to be determined in advance but updated by the adaptive rules. Sufficient conditions are provided to guarantee the stable synchronization in the sense of the Lyapunov stability. In addition, case study and numerical simulations are performed to verify the effectiveness of proposed schemes.

Key words. Robust adaptive, two-level sliding mode, single input control, state synchronization.

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[2]Workshop 1 - Department of Computer-Aided Industrial Design Overseas Chinese University, Taichung City, Taiwan

[3]Workshop 2 - Department of Mechanical Engineering, National Yunlin University of Science & Technology, Touliu, Yunlin, Taiwan

[4]Workshop 3 - Nindge Normal University, Fujian, China

[5]Workshop 4 - Department and Graduate School of Electrical Engineering, Hsiuping University of Science & Technology, Taichung City, Taiwan.

[6]Corresponding author: Chiching Yang; e-mail: ccyang@hust.edu.tw

1. Introduction

Chaos in many systems is a source of the generation of oscillation. Chaotic synchronization exists in various fields of applications such as economic control system [1], internet congestion control [2], and secure communication [3]. Since the results of synchronizing two identical chaotic systems from different initial conditions was firstly introduced in 1990 [4], the relative researching topics were attracted many attentions.

The idea of chaotic synchronization of states is to use the states of the drive system to control the driven system so that the states of the driven system follow the states of the drive system asymptotically. For the state synchronization of n-dimensional chaotic systems with single control input, there were many studies to be addressed, for example, [5-11] and the references therein. Synchronization the Genesio-Tesi type chaotic systems with muti-control inputs were introduced in [12-14]. However, in these studies [12-14], the redundant control inputs for synchronization are not necessary and cause the structure of control too complicated. In fact, control problem of chaotic synchronization mentioned in [12-14] can be accomplished by only single input control.

In this study, the novel two-level sliding mode control is introduced to achieve the state synchronization between two the same kind of n-dimensional chaotic systems in the presence of system uncertainty and external disturbance. The proposed robust adaptive control law is not required the known bounds of system uncertainties and external disturbances in prior. The proposed robust adaptive control law associated with time varying feedback gains can compensate nonlinear dynamics of the synchronous error system. Meanwhile, these feedback gains are not to be determined in advance but updated by the adaptive rules. In the sense of the Lyapunov stability theorem, sufficient conditions are provided to guarantee the stable synchronization. In addition, numerical study of two the same kind of three dimensional Genesio-Tesi systems is given to verify the effectiveness of proposed schemes.

The rest of the study is organized as follows. State synchronized control problem of n-dimensional nonlinear chaotic system is proposed section 2. In Section 3, the concept of two level sliding mode control design is introduced and the procedures of robust adaptive control law design is provided and proved. In Section 4, case study and numerical simulations of two the same kind of three dimensional Genesio-Tesi systems are performed to verify the effectiveness of presented scheme. In the final section, the conclusions are made.

2. Formulation of the state synchronized control problem

Consider the following n-dimensional nonlinear chaotic system described by:

$$\begin{cases} \dot{z}_i = z_{i+1},\ 1 \le i \le n-1 \\ \dot{z}_n = F(Z(t)) \end{cases} \tag{1}$$

Where $Z(t) = \begin{bmatrix} z_1(t) & z_2(t) & \cdots & z_n(t) \end{bmatrix}^T$ is state vector. It is assumed that the scalar function $F(Z(t))$ is satisified the global Lipschitz condition. That is,

$$|F(Z(t))| < \rho_0 + \sum_{i=1}^{n} \rho_i |z_i| \tag{2}$$

Where$\rho_i > 0$, $i = 0, 1, \cdots, n$. In the literature, there are many published chaotic systems belong to the proposed class, such that, Genesio-Tesi system [15] and Arneodo's system [16]. In the following, the synchronization between two n-dimensional the same kind of chaotic systems in (1) are considered, even when the driven system with single control input is undergoing system uncertainties and external disturbances. First of all, the drive system is defined in (1) and the driven system is described as follows,

$$\begin{cases} \dot{x}_i = x_{i+1}, \ 1 \le i \le n-1 \\ \dot{x}_n = F(X(t)) + \Delta f(X(t)) + u(t) + \Delta(t) \end{cases} \tag{3}$$

Where $X(t) = \begin{bmatrix} x_1(t) & x_2(t) & \cdots & x_n(t) \end{bmatrix}^T$ is the state vevtor, $\Delta f(X(t)) \in R$ is the bounded system uncertainty satisfying $0 < |\Delta f| < B_f$, $\Delta(t) \in R$ is the external disturbance satisfying $0 < |\Delta(t)| < B_d$, and $u(t)$ is the control input.

The synchronous error states between systems (1) and (3) are defined as follows:

$$e_i(t) = x_i(t) - z_i(t), \quad i = 1, \ \cdots, n. \tag{4}$$

Then, the problem of state synchronization between systems (1) and (3) can be defined. By taking the time derivatives of (4) along with (1) and (3), the synchronous error system can be expressed as

$$\begin{cases} \dot{e}_i = e_{i+1}, \ 1 \le i \le n-1 \\ \dot{e}_n = F(X(t)) - F(Z(t)) + \Delta f + u(t) + \Delta(t) \end{cases} \tag{5}$$

To this end, it is clear that the problem of state synchronization is replaced by the equivalence of stabilizing the synchronous error system (5) by applying a suitable design of the control law $u(t)$. The control goal for the current problem is to determine the appropriate control law $u(t)$ such that for any initial states of the synchronous error system (5), states of the synchronous error system converges to zeros, that is, $\lim_{t\to\infty} e_i(t) \to 0, \ i = 1, \cdots, n$.

3. Design of Robust-adaptive Sliding mode Control Law

In the following, the rule of thumb for the robust-adaptive synchronized control design is introduced. Two basic steps are involved in order to accomplish the state synchronization between systems (1) and (3). First, two levels of sliding function are chosen. Then, the adaptive and robust control law is designed for that any trajectory in phase space of the synchronous error system can be brought to and stayed in the sliding surface even in the presence of system uncertainty $\Delta f(X)$ and external disturbance $\Delta(t)$.

The concept for two levels sliding functions is introduced as follows. The level 1 sliding surface $s(t)$ is the function of n-dimensional error states $e_i(t),\ \ i=1,\ \cdots,n$, where the desired sliding motion is embedded. The level 2 sliding surface $\sigma(t)$ is formed with the level 1 sliding function $s(t)$.

The level 1 sliding surface $s(t)$ is chosen as follows.

$$s(t)=e_n(t)+\int_0^t\sum_{i=1}^{n}c_ie_i(\tau)d\tau \tag{6}$$

Where $c_1,\ c_2,\cdots,c_n$ are design parameter, which satisfying the polynomial $\lambda^n+c_n\lambda^{n-1}+\cdots+c_1=0$ is Herwitz. When the states of the synchronous error system approaching to and staying in the sliding surface, it means that conditions $s(t)=0$ and $\dot{s}(t)=0$ are satisfied. $\dot{s}(t)=0$ can be expressed as

$$\begin{array}{l}\dot{e}_i=e_{i+1},\ 1\leq i<n\\ \dot{e}_n(t)=-\sum_{i=1}^{n}c_ie_i(t)\end{array} \tag{7}$$

(7) can be represented further

$$\dot{E}(t)=AE(t) \tag{8}$$

Where $E(t)=\begin{bmatrix}e_1(t) & e_2(t) & \cdots & e_n(t)\end{bmatrix}^T$ is the error vector and A is the companion matrix with the coefficients $c_1,\ c_2,\cdots,c_n$.

The level 2 sliding surface $\sigma(t)$ is defined

$$\sigma(t)=[s(t)]^{p/q}+k\int_0^t s(\tau)d\tau \tag{9}$$

Where $k>0$ and $1<p/q<2$, p,q are positive odd integers. For $\sigma(t)=0$ and $\dot{\sigma}(t)=0$, it can be yield that

$$\dot{\sigma}(t)=ks(t)+\frac{p}{q}[s(t)]^{(p/q)-1}\dot{s}(t)=0 \tag{10}$$

For the time $t_{\mathrm{r}}\geq 0$ of the level 1 sliding function $s(t)$ from initial values reaching to the terminal sliding surface$\sigma(t)=0$, the finite time t_{s}, that is taken to travel from $s(t_{\mathrm{r}})\neq 0$ to $s(t_{\mathrm{r}}+t_{\mathrm{s}})=0$, is given by

$$t_s=t_r+\frac{p}{k(p-q)}[s(t_r)]^{(p/q)-1} \tag{11}$$

To this end, it is concluded that the object of control law design is to force $\sigma(t)=0$ and $\dot{\sigma}(t)=0$. Then, conditions $s(t)=0$ and $\dot{s}(t)=0$ are satisfied such that n-dimensional error states $e_i(t),\ \ i=1,\ \cdots,n$ tend to zeros.

Theorem.

If the control law $u(t)$ in system (3) is applied as follows:

$$u(t) = -(kq/p)\,[s(t)]^{2-p/q} - \left[K_0(t) + \sum_{i=1}^{n} K_i(t)\,|e_i(t)|\right] \cdot \mathrm{sign}(\sigma(t)) \qquad (12)$$

Where the $s(t)$ and $\sigma(t)$ are defined in (6) and (9), respectively. sign($\bullet$) is the sign function. The positive and adaptive feedback gains, $K_0(t)$ and $K_i(t),\ i = 1, \cdots, n$ are updated according to the following adaptation algorithms, respectively,

$$\dot{K}_0(t) = \gamma_0\,|\sigma(t)|\,|s(t)|^{(p/q)-1} \geq 0, \quad k_0(0) = 0 \qquad (13)$$

$$\dot{K}_i(t) = \gamma_i\,|e_i(t)|\,|\sigma(t)|\,|s(t)|^{(p/q)-1} \geq 0, \quad k_i(0) = 0 \qquad (14)$$

Where $\gamma_i,\ i = 0, 1, \cdots, n$ are the positive constants determining the adaptation process. Then, the level 1 sliding function $s(t)$ will asymptotically approach to and stay in the terminal sliding surface $\sigma(t) = 0$, and then converge to zeros in finite time. It follows that states of the synchronous error system (7) will asymptotically approach to and stay in the sliding surface $s(t) = 0$. Then, all states of the synchronous error system converge to zeros and the state synchronization between systems (1) and (3) can be achieved.

Proof.

The Lyapnnov candidate function is selected to be

$$V(t) = \frac{1}{2}\sigma^2(t) + \frac{p}{q}\sum_{i=0}^{n}\frac{1}{2\gamma_i}(K_i(t) - \bar{K}_i)^2 \geq 0 \qquad (15)$$

Where $\bar{K}_i,\ i = 0, 1, \cdots, n$ are sufficient large positive constants and satisfy the following inequalities

$$\begin{array}{l} \bar{K}_0 > \rho_0 + B_f + B_d > 0, \\ \bar{K}_i > \rho_i + c_i > 0,\ i = 1, \cdots, n. \end{array} \qquad (16)$$

Taking the time derivative of (15) along with the solutions of the synchronous error system (7), the selection of the two level sliding functions (6) and (9), and the control law (12), it obtains

$$\begin{array}{l} \dot{V} = \sigma\dot{\sigma} + \frac{p}{q}\sum_{i=0}^{n}\frac{1}{\gamma_i}(K_i - \bar{K}_i)\dot{K}_i \\ \quad \leq \sigma\left[ks + (p/q)[s]^{(p/q)-1}\left(\rho_0 + \sum_{i=1}^{n}\rho_i\,|e_i| + \sum_{i=1}^{n} c_i\,|e_i| + \Delta f + \Delta + u\right)\right] \\ \quad + \frac{p}{q}\sum_{i=0}^{n}\frac{1}{\gamma_i}(K_i - \bar{K}_i)\dot{K}_i. \end{array} \qquad (17)$$

By substituting (13) to (14) into (17) and with the criterions in (16), i14t yields

$$\dot{V} < \frac{p}{q}\,|s|^{(p/q)-1}\left[-(\bar{K}_0 - \rho_0 - B_f - B_d)\,|\sigma| - \sum_{i=0}^{n}(\bar{K}_i - \rho_i - c_i)\,|e_i|\,|\sigma|\right] < 0 \qquad (18)$$

Therefore, the condition for Lyapunov stability is satisfied. The level 2 sliding function $\sigma(t)$ can reach to $\sigma(t) = 0$ asymptotically and stay in it. Then, the level 1 sliding function $s(t)$ is approaching to zero in the finite time according to (11). It is induced that the synchronized error states tend to zeros asymptotically.

4. Case study and numerical simulations

In this section, the effectiveness of the proposed adaptive robust control law is verified. The numerical simulations for two the same kind of three dimensional Genesio-Tesi systems [15] are performed. In numerical simulations, the fourth-order Runge-Kutta method is used to solve the system with time step size of 0.0001.

The Genesio-Tesi system has the form represented as follows

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = z_3 \\ \dot{z}_3 = -cz_1 - bz_2 - az_3 + z_1^2 \end{cases} \tag{19}$$

Where the system parameters are $a = 1.2,\ b = 2.92,\ c = 6$ can perform the chaotic phenomena [15]. For two Genesio-Tesi systems (1) and (3), the synchronous error system can be expressed as

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = e_3 \\ \dot{e}_3 = -ce_1 - be_2 - ae_3 + \phi(x_1,\ z_1)e_1 + \Delta f(x_1,\ x_2,\ x_3) + \Delta(t) + u(t) \end{cases} \tag{20}$$

The initial conditions of the drive and the driven systems are chosen $(z_1(0),\ z_2(0),\ z_3(0)) = (1,\ -1,\ 1)$ and $(x_1(0),\ x_2(0),\ x_3(0)) = (2,\ 1,\ 3)$, respectively. For the adaptive robust control law (12) associated with (6), (9), (13), and (14), the positive constants are chosen as $k = 2,\ p = 9,\ q = 7,\ c_1 = 1,\ c_2 = 3,\ c_3 = 2.5$ $\gamma_0 = \gamma_1 = \gamma_2 = 0.025,\ \gamma_3 = 0.015$. It is supposed the system uncertainty is assumed $\Delta f = \sin(x_1)\sin(x_2)\sin(x_3)$ and the external disturbance is $\Delta(t) = \cos(t)$ in the following numerical simulations.

Time responses of the synchronous error states are depicted in Figure 1. Figure 2 and 3 exhibit the time responses and the trajectories for the two kinds of sliding functions, respectively. It is shown both $s(t)$ and $\sigma(t)$ converge to zero and the state synchronization is achieved. In Figure 4, it is demonstrated that the applied control input is continuous and chattering free in the presence of system uncertainties and external disturbances. Time response of adaptive feedback gains are shown in Figure 5. Figure 6 displays time responses of the drive and driven system states, respectively. It is obviously depicted that state synchronization is accomplished.

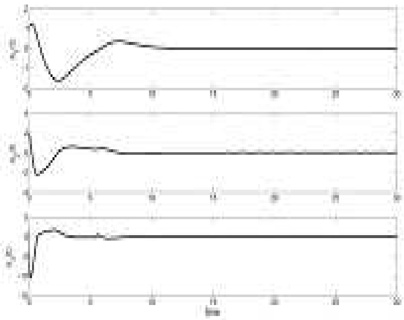

Figure 1. Time responses of synchronous error states.

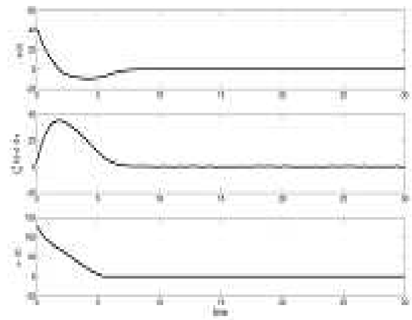

Figure 2. Time responses for the two levels of sliding functions.$s(t)$, $\sigma(t)$.

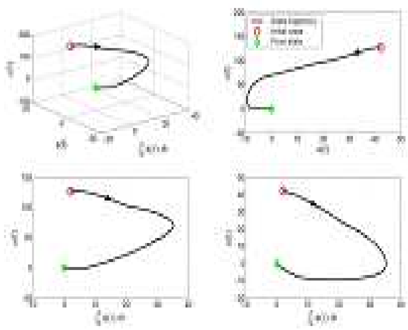

Figure 3. Trajectories for two levels of sliding functions.

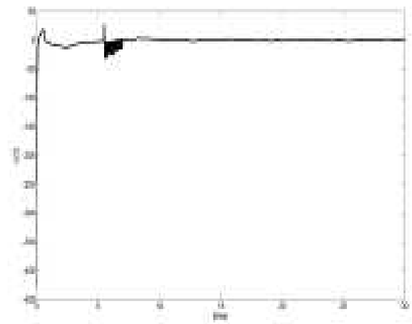

Figure 4. Time responses of of the control input.

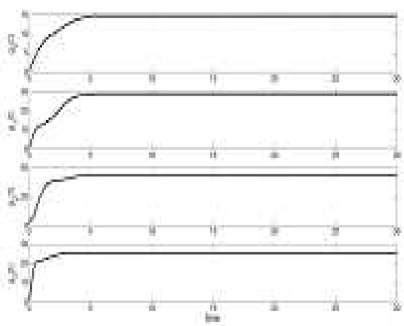

Figure 5. Time responses of adaptive feedback gains.

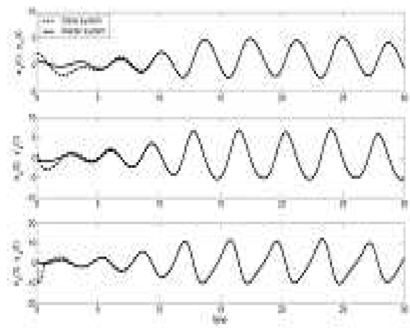

Figure 6. State trajectories of two synchronized Genesio-Tesi systems.

5. Conclusions

In this study, for achieving state synchronization between two the same kind of n-dimensional chaotic systems belong to system class (1) in the presence of system uncertainties and external disturbances, the robust adaptive sliding mode control law has been addressed. The implementation of the proposed adaptive control law is not required the known bounds of system uncertainties and external disturbances. The proposed adaptive control law associated with time varying feedback gains can compensate nonlinear dynamics of the synchronous error system. Meanwhile, these feedback gains are not to be determined in advance but updated by the adaptive rules. In the sense of the Lyapunov stability theorem, sufficient conditions are provided to guarantee the stable synchronization. In addition, numerical study of two the same kind of three dimensional Genesio-Tesi systems are given to verify the effectiveness of proposed schemes.

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